JPL PUBLICATION 80-44

(NASA-CR-163438) HISTORY OF THE MASS OF MERCURY (Jet Propulsion Lab.) 36 p HC A03/MF A01 CSCL 03B

N80-30261

Unclas

G3/91 28412

History of the Mass of Mercury

R.A. Lyttleton

August 1, 1980

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

JPL PUBLICATION 80-44

History of the Mass of Mercury

R.A. Lyttleton

August 1, 1980

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under NASA Contract No. NAS7-100.

ABSTRACT

Since 1895 the mass of Mercury has been generally accepted as 6000000^{-1}. The mean density implied is extremely high, but it would be consistent with the other terrestrial planets for mass near 9000000^{-1}. Attempts to refine the mass value involve nonlinear (differential) equations in many variables, and for proceeding to least-squares there must be introduced starting-values already near the true values for the results to be reliable. If a particular value is widely wrong, only a small formal correction to it will emerge.

The masses of Mercury and Venus were very uncertain before Newcomb presented his extensive study. For Venus this gave 408000^{-1}. For Mercury he omitted to complete the calculation, which would have given 8405291^{-1}. But later on, without explanation, Newcomb "took" 6000000^{-1}, thereby rejecting the value resulting from his investigation. This baseless figure 6000000^{-1} somehow became generally accepted and adopted in subsequent attempts to improve the mass. That only small changes have emerged is likely to be no more than a consequence of relying on a least-squares process.

Reduction of the Mercury mass to 9000000^{-1} would alter theoretical angular positions of Venus by less than inherent observational errors, but the theoretical distances would be changed by amounts probably capable of detection by radar means.

CONTENTS

Figures

Tables

SECTION I

MODERN ACCEPTED VALUES FOR THE PLANETARY MASSES

In order to construct reliable Tables for the positions of the planets at any time, the values of their masses need to be known with all possible accuracy in terms of some unit of mass. Absolute values (in grams for example) are not essential, and the standard generally made use of is the ratio of the mass of the Sun to that of the planet, usually referred to as the reciprocal mass. Herein we shall be concerned almost entirely with the four inner planets and the Moon, though in the earliest part of the history the masses of Jupiter and Saturn enter, as will later be seen.

For several decades, the values generally agreed upon for the planetary masses and for the Earth-Moon mass ratio have been those given in the Explanatory Supplement to the Nautical Almanac (Reference 1-1) first published in 1961, and reprinted unchanged in the third impression dated 1974. The values given that are relevant to the present discussion are those in Table 1-1. The values for Mercury and Venus, which are of prime interest in this history, remain precisely those taken originally by Newcomb in 1895.

Table 1-1. 1961 Values for Planetary Masses

Planet	Value	Planet	Value
Mercury	6 000 000	Earth	333 432
Venus	408 000	Mars	3 093 500
Earth+Moon	329 390	Jupiter	1 047.355
Earth-Moon ratio	81.45	Saturn	3 501.6

At the Twelfth General Assembly of the I.A.U., Hamburg 1964, (Reference 1-2), practically the whole of the mass values of Table 1-1 were again endorsed as representing the I.A.U. System of Astronomical Constants, except that the reciprocal mass of the Earth was changed to 332958, and that of the Earth+Moon, as a derived secondary constant, was changed to 328912 (p. 595, item 19), though in the System of Planetary Masses given lower down on the same page, the value for Earth+Moon is left unchanged (from the 1961 value) at 329390. On these revised values, the implied Earth-Moon mass ratio is 81.2935.

The advent of space missions held promise of opportunities for refinement of these mass values, and at the Sixteenth General Assembly of the I.A.U., Grenoble 1976 (Reference 1-3), the revised values given in Table 1-2 were endorsed as thenceforth representing the I.A.U. System of Planetary Masses (the figures in parentheses are the proportionate changes).

Table 1-2. I.A.U. System of Planetary Masses: 1976

Planet	Value	Factor of Change	Planet	Value	Factor of Change
Mercury	6 023 600	(1.0039)	Earth	332 946	(0.9985)
Venus	408 523.5	(1.0013)	Mars	3 098 710	(1.0017)
Earth+Moon	328 900.5	(0.9985)	Jupiter	1 047.355	(1.0)
Earth-Moon ratio	81.30	(0.9982)	Saturn	3 498.5	(0.9991)

It is seen that where the terrestrial planets and the Moon are concerned, in this interval of just over a decade, adjustments have been made of between 0.1 and 0.2 percent, except for Mercury for which the change introduced is by nearly 0.4 percent.

The mass values of Table 1-2 have been made use of, as quantities requiring no further adjustment, by Anderson et al. 1976 (Ref. 1-4) in making a determination of certain relativity parameters, the solar quadrupole moment, and other extremely small quantities associated with the solar system.

SECTION II

THE ANOMALOUS MEAN DENSITY OF MERCURY COMPARED WITH THE OTHER INNER PLANETS AND THE MOON

The mean density of Mercury implied by these masses is found to be highly anomalous compared to the other members of the terrestrial group and the Moon. To demonstrate the extent of this, Table 2-1 shows the present values for the masses and mean densities of the several bodies, where the mass of the Moon is adopted as the unit, and the currently accepted radius for Mercury of 2.44×10^8 cm is adopted.

Table 2-1. Masses and Densities of the Inner Planets

Planet	Mass	$\log_e$mass	Mean Density, g cm^{-3}
Moon	1.00	0.0	3.35
Mercury	4.49	1.502	5.41
Venus	66.24	4.193	5.23
Earth	81.30	4.398	5.52
Mars	8.73	2.167	3.92

The advantage of using the logarithm of the mass is that it leads to a more compact diagram than if the mass itself were used. Figure 2-1 is a graph of the mean density against $\log_e$mass for the five bodies. It is seen at once that although a remarkably smooth curve can be drawn through the points representing the Earth, Venus, Mars, and the Moon, the point for Mercury lies a great way off the curve, and even the present simple considerations strongly suggest that there is something not fully understood about the planet.

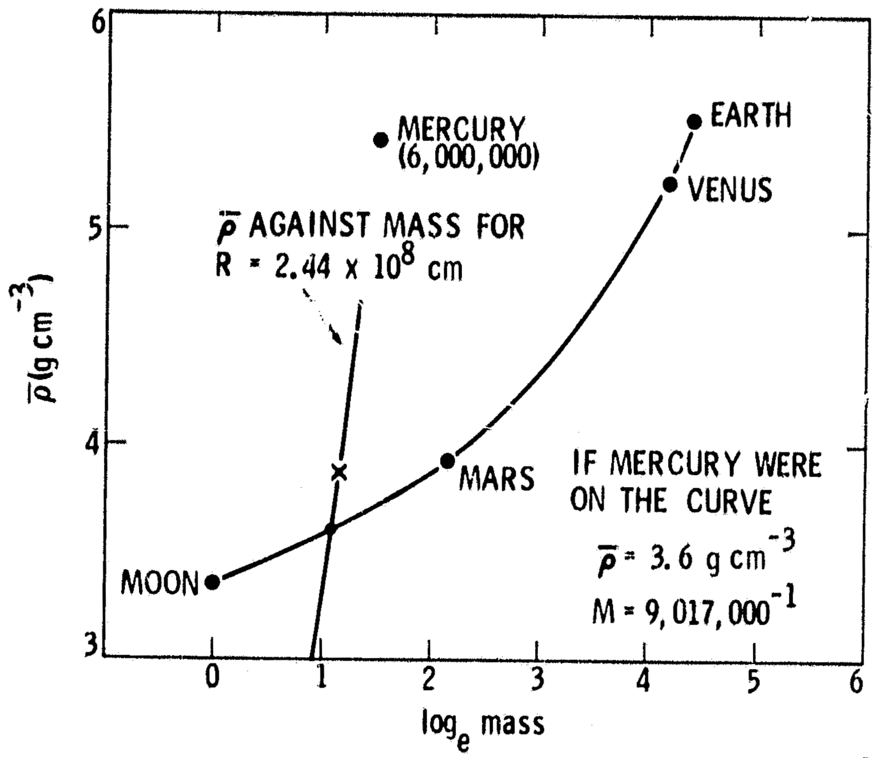

Figure 2-1. Plot of Mean Density vs $\log_e$Mass for the Inner Planets and the Moon (=1)

SECTION III

RECENT CLAIMS FOR THE SOURCES OF THE VALUES USED FOR THE MASS OF MERCURY

The value 6000000 for the reciprocal mass of Mercury has been maintained by Kuiper, 1970 (Ref. 2-1), to have first emerged from work by Rabe in 1950 (Ref. 2-2) based on observations of Eros, though in his paper Rabe ascribes the value to Stracke, 1940 (Ref. 2-3). However, in this latter paper, Stracke simply adopts 6000000 without giving any source. In 1967, Rabe himself announced that his 1950 work was entirely vitiated by a conceptual error made at the outset (Ref. 2-4). With this put right, and using 5700000 as starting-value, Rabe concluded that the value of the mass was almost completely indeterminate from the perturbations of Eros for 1926-45, and he gave a calculated formal value of 4600000 ± 2500000 for the reciprocal mass of Mercury. A number of more recent authors have also adopted a mass near 6000000 as a starting-value for least-squares adjustment. Thus, Ash, Shapiro, and Smith, 1967 (Ref. 2-5), used 6110000 as initial value, a figure obtained by Clemence in 1965 (Ref. 2-6) using precisely 6000000 as a starting-value for slight correction. Brouwer and Clemence in 1961 (Ref. 2-7) claimed, quite correctly, that the conventionally adopted mass of Mercury is $m^{-1} = 6000000$, but go on to state that it was "obtained by Newcomb in 1895 (Ref. 2-8) from a general adjustment of quantities affecting the motions of the four inner planets." It will be seen later, however, that this latter statement by no means correctly represents the actual situation, in that nowhere did Newcomb actually derive the value 6000000.

H. N. Russell, 1945 (Ref. 2-9), in discussing the Mercury mass, points out that only approximate values can be found because the disturbing effects of Mercury on other planets are so slight. In his Table IV of Elements and Constants, Russell gives the value 7500000^{-1}, but notes it as uncertain, and by way of illustration remarks that Newcomb at one time used 7500000^{-1} (Newcomb's original starting value in fact), while at another gave 6000000^{-1}, but Russell does not mention any value emerging from the lengthy investigation by Newcomb in the

1880s and early 1890s (Ref. 2-8). Russell also states that de Sitter assigned a value of 8000000^{-1} to the mass, but the present writer has been unable to locate any paper by de Sitter confirming this. Indeed, it would appear that in his own studies of the system of astronomical constants de Sitter adopted 7500000 (1 ± 0.20), and Brouwer, 1938 (Ref. 2-10), states that this mass is the value used by Newcomb.

SECTION IV

EFFECT OF NONLINEARITY OF THE EQUATIONS THAT DETERMINE PLANETARY MASSES

It is important to remember that the dynamical equations governing planetary motions are highly nonlinear. Hence in applying them to a large number of observations they can be reduced to manageable form only by developing them to a stage when some process such as that of the method of least-squares can be applied, which means, of course, the need for linearized equations. But for such a purpose, starting-values of the quantities sought for are required, and it is implicitly assumed that those made use of are already close to the true values. But if for some particular quantity a starting-value far from the true value should chance to be adopted, then the standard procedure of least-squares may result in no more than a small formal correction to it, not necessarily even in the right direction, and especially could this happen when a large number of variables are simultaneously involved for small corrections. If the starting-value for one particular variable is seriously wrong, the computed corrections to the others may all carry a part resulting from the large error in the particular quantity while leaving its adopted value only slightly changed. To overcome this, it would be necessary to make the whole least-squares calculation for a series of different starting-values for the suspect quantity, and examine how the resulting sum of squares of the residuals is affected along the series, with a view to determining the "best" value.

SECTION V

CONJECTURAL EXPLANATIONS OF THE HIGH MEAN-DENSITY OF MERCURY

A number of verbal explanations have been proposed for the alleged high mean-density of Mercury, which is almost that of the Earth, as for example that it is a natural consequence of the proximity of the planet to the Sun, and therefore only to be expected. Yet Venus also is closer to the Sun than is the Earth, but its internal structure shows not the slightest indication of any higher-temperature effect on its composition. Another suggestion has been that the solar nebula from which the planets accumulated had higher-density substances in its inner annular region, where Mercury presumably formed, to a much greater extent than elsewhere. This, however, is much too facile to be a satisfactory explanation, reminiscent as it is of "Things are as they are because they were as they were." (Gold). It would require that a narrow innermost annulus of the dust disc, with less than 3 percent of the combined mass of the inner planets, had essentially different composition with a much higher proportion of heavy elements than the remaining 97-plus percent. But if the original gas-dust nebula were captured, its materials would be well mixed, and this would therefore seem a quite unacceptable postulate. Both suggestions would run into difficulty if the core of the Earth, carrying over 31 percent of the whole mass, were interpreted as nickel-iron. For then the core of Venus would contain just under 25 percent of the planet as nickel-iron (to give the correct radius), which would mean a <u>decrease</u> of heavier material nearer the Sun, not an increase.

All in all, it has long seemed a situation very difficult of reconciliation with any theory of the origin of the terrestrial planets, and even more so with the theory of planetary structure. The mean density of Mercury, with the current values of the mass and radius, would be as high as 5.41 g cm^{-3}, only slightly less than that of the Earth, and to account for this by means of high iron-content would require the planet to consist of about 60 percent iron and the remaining 40 percent rocky material (Reference 5-1). Curiously enough the observed

surface properties of Mercury show no indication of any unusuality of composition, but closely resemble those of the Moon. On the phase-change hypothesis for the nature of the terrestrial core, iron ceases to have high abundance, and this accords with the mean densities of Mars and the Moon. At the mantle-core boundary in the Earth, the phase-change occurs at present at a pressure of 1.37×10^{12} dyn cm^{-2}, and there can be no possibility of the central pressure in Mercury being as high as even one-third this value. Thus the phase-change cannot be invoked for Mercury, and the high density could therefore only be explained by this large iron-content, which would obviously render it a strangely exceptional object among the terrestrial group of bodies. The currently accepted value of the radius, 2.44×10^{8} cm, seems unlikely to be seriously in error, and thus attention focusses on the mass if the structure of Mercury is to conform with the other terrestrial planets. Only after several years of serious misgivings on the part of the writer about this difficulty was it suddenly perceived to be essential to go over the whole history of the mass of Mercury to determine just at what point in time the value 6000000^{-1} first surfaced in the literature, and of course exactly how it came to do so. The outcome of the resulting investigation has led to a most surprising discovery.

SECTION VI

THE EARLIEST ATTEMPTS TO DETERMINE THE MASSES OF MERCURY, VENUS, AND MARS

The discovery, yet to be described as to its nature, was made as a result of proceeding right back in time through all the numerous efforts to determine the mass of Mercury. These began late in the 18th century when the theory of the motions of the planets was being developed by Laplace (Reference 6-1) and many others. At that period, the Martian satellites remained undiscovered, and thus there was no simple way to find even approximately the mass of Mars, and similarly of Venus and Mercury, in terms of that of the Sun. To meet this difficulty, Lagrange introduced the purely conjectural hypothesis that the mean density of any planet is inversely proportional to its distance from the Sun, a rule that found some support from the values for the Earth, Jupiter, and Saturn, which possessed one or more satellites. The angular diameters of the planets, which are needed to find the densities, were known at that time only with moderate accuracy, but the present values used for Table 6-1 adequately illustrate Lagrange's idea.

Table 6-1. Lagrange's Rule for Earth, Jupiter, and Saturn

Parameter	Earth	Jupiter	Saturn
Distance, AU	1.00	5.20	9.54
Density, g cm^{-3}	5.52	1.334	0.684
Product	5.52	6.94	6.63

The agreement is seen to be no more than fair, but in the complete absence of any other guide, Laplace extended the rule to the three planets Mercury, Mars, and Jupiter, for only the last of which were both the distance and density known. Again using present values, so that the product of distance and density would be 6.94 units, the rule would give the values shown in Table 6-2. The resulting density for Mars is seen to be only about one-sixth too great, but that for Mercury

Table 6-2. Estimates of Density for Mercury and Mars

Parameter	Mercury	Mars
Distance, AU	0.3871	1.5237
Implied density, g cm^{-3}	17.93	4.56

is clearly almost impossibly on the high side. Despite this, Laplace soldiered on, and the resulting values of the masses, in terms of that of the Sun (using the presently known values of the diameters), would be:

$$\text{Mercury} \quad 1823080^{-1} \qquad \text{Mars} \quad 2675782^{-1}$$

Laplace in fact obtained, and used in his planetary theory, the following somewhat different values:

$$\text{Mercury} \quad 2025810^{-1} \qquad \text{Mars} \quad 1846082^{-1}$$

The first of these is about three times the mass currently adopted for Mercury, while the second is about five-thirds the current value for Mars.

The next estimate of the Mercury mass was given by Encke in 1841 (Reference 6-2), and was based on a study of the perturbations in the orbit of the short-period comet, now named after him, for the interval 1832-1838. Encke arrived at the result 4865751^{-1} for the mass of Mercury.

In 1842, Rothman (Reference 6-3) obtained a value 3182843^{-1} from the part produced by Mercury in the motion of the perihelion of Venus. As yet, LeVerrier, 1859 (Ref. 6-4), was still to demonstrate that the perihelion of Mercury exhibited an entirely inexplicable residue in its motion, while, in addition, the orbit of Venus has such small eccentricity that the motion of its perihelion must itself have been extremely uncertain of determination.

LeVerrier first came on the scene in the present connection with his Tables of Venus, 1844 (Ref. 6-5), in which, for the perturbations of Venus and the Earth, he made use of the round figure 3000000^{-1} for the mass of Mercury. Seventeen years later, from the perturbations of Venus by Mercury, LeVerrier found the mass to be 5310000^{-1}, and, from the perturbations of the motion of the Earth, 4360000^{-1} (Reference 6-6).

Lastly, in this era of the history, von Asten in 1876 (Reference 6-7) arrived at the value 7636440^{-1} from analysis of the perturbations of Comet Encke during 1819-1868, a period far more extensive than that available to Encke himself in 1841.

Then in 1877, the satellites of Mars were discovered, and henceforth the mass of this planet could be regarded as known with far greater certainty.

From this improved situation, Tisserand in 1881 (Reference 6-8), from a discussion of the perturbations of Venus, obtained the value 7100000^{-1} for the Mercury mass. Several years later, in his famous Mécanique Céleste, Tisserand was of the opinion that the value close to 9700000^{-1}, obtained by Backlund from his study of the motion of Encke's comet, merited serious consideration.

Next, in 1889, a whole series of five different values were announced by von Haerdtl (Reference 6-9). First, using perturbations of Comet Encke over the interval 1858-1886, he found 5514700^{-1}. Second, using results taken from von Asten for the motion of comet Encke, von Haerdtl found 5648600^{-1}; and third, using results of Backlund for comet Encke, he found 5669700^{-1}. A fourth result was reached by a discussion of four equations obtained by LeVerrier that yielded 5514700^{-1}. Lastly, von Haerdtl reconsidered an equation of LeVerrier and included an observation made by Horrocks in 1639 (presumably related to the transit of Venus in that year), which it seems LeVerrier had rejected, and this led to mass 17120000^{-1}. All the material made use of predated the discovery of Phobos and Deimos, and thus involved a highly uncertain mass of Mars.

In 1885 Backlund discussed the perturbations of comet Encke for the interval 1871-1885 and concluded with a mass-value of 2668700^{-1} for Mercury, but in 1894 the same author (Reference 6-10), from the perturbations of the comet over the longer interval 1871-1891, found a value of 9647000^{-1}. The great difference shows the large uncertainty of any result based on this comet.

Going forward a few years, G. W. Hill (of lunar theory fame), perhaps despairing of the large discordances of masses arrived at by dynamical means, in 1898 (Reference 6-11) reverted to a physical approach (Reference 6-12) that made use of an extended form of the internal-density law of Laplace. Suffice it here to state that Hill arrived at a value 11634200^{-1} for the Mercury mass, while after some adjustments "allowing for the cooling of the planet since its formation", the mass was increased slightly to 10530500^{-1}.

SECTION VII

THE EXTENSIVE INVESTIGATION BY NEWCOMB OF THE MOTIONS OF THE FOUR INNER PLANETS

Returning now to the successive dynamical determinations of the mass of Mercury, in the 1880s the renowned Simon Newcomb, then responsible for producing the American Ephemeris, commenced a thoroughgoing investigation of the motions of the four inner planets. This was based upon 62,030 meridian observations of the Sun, Mercury, Venus, and Mars made at thirteen observatories, spaced worldwide. The observations from Greenwich covered the complete period 1750-1892 under discussion, the next largest contribution was from Paris for the years 1801-1889, while the shortest was from Strassburg 1884-1887. Further to this fundamental material, Newcomb had available the far more reliable data for Venus emerging from the four transits of the planet that occurred in the years 1761, 1769, and 1874, 1882. In addition, Newcomb also had at his disposal the similarly more reliable data for Mercury derived from 23 transits of the planet observed during the time range 1677-1881. Publication of this massive investigation, stage by stage, was to fill many pages of the journal founded by Newcomb in the early 1880s entitled "Astronomical Papers prepared for the use of the American Ephemeris." At the very beginning of this work, which occupied several years, Newcomb lists for the provisionally accepted mass of Mercury the value found by Encke as long before as 1841, namely 4865751^{-1}, though Newcomb does not seem to have made any actual use of it.

When the work was completed, Newcomb in 1895 published the whole in summary, but by no means brief form, which described all the procedures adopted and the main stages and steps of the calculations. The resulting volume, which has since become classical, was entitled The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy (Reference 2-8).

In this, Newcomb explains first that if the secular variations of the elements arose solely from the actions of the planets upon each other, it would be possible to derive the masses of the planets from

these perturbations. But LeVerrier had long since been the first to demonstrate that the whole advance of the perihelion of Mercury was inexplicable by planetary action only, as also but to a lesser extent was the motion of the node of Venus. At that time, it was suspected there might be unrecognized masses near the Sun: indeed, discovery of an intra-Mercurial planet was actually reported in 1859, when it was given the name Vulcan, and again in 1878, but it is as near certain as possible that no such object more than 20 km in radius can be present. (Reference 2-9, p. 358.) Accordingly, Newcomb was forced to make use of the periodic inequalities to determine the masses of Venus and Mercury.

The greatest perturbation produced by Mercury on any planet is in the longitude of Venus, and it has a coefficient of only 0".361 (for Mercury mass 6000000^{-1}) and a period of 5.662 years. The next largest has a coefficient of 0".265 and a period of 1.110 years, while the next has a coefficient of 0".105 and a period of 0.555 year. Thus for the largest, one is seeking a term of total range less than 0".8 but of accurately known period. However, measures of the angular position of the center of Venus are subject to considerable inaccuracy (for well-known reasons) with a probable error of ±1".0 in both R.A. and Declination. On the other hand, transits of Venus across the solar disc enable positions of the planet to be determined with far greater accuracy. Such a transit occurred in 1639, but was observed only in England, and not used by Newcomb, but four more occurred in the years 1761 and 1769, and 1874 and 1892, which he did use. Newcomb accordingly decided to make two separate solutions: first, a solution (A) based solely on the meridian observations, and secondly, a solution (B) obtained by including also the results from transit observations of both Venus and Mercury. The latter each gave six resulting equations of condition that Newcomb judged to have weights 250, 300, 400, 700, 700, and 1600 in the case of Mercury, and weights 200, 400, 800, 200, 600, and 1600 for the six equations for Venus-transits, the unit-weight being that of a single meridian observation of Venus.

ORIGINAL PAGE IS
OF POOR QUALITY

SECTION VIII

NUMERICAL RESULTS DERIVED BY NEWCOMB FOR THE PLANETARY MASSES

Before giving Newcomb's results for Mercury, it is of interest that, starting with the value 327000^{-1} for the mass of the Earth+Moon, the final value obtained was 328016^{-1} with a probable error of "something more than a thousandth part of its whole amount." Compared with the 1976 value given earlier herein, this value is only 0.27 percent greater. For the mass of Venus, the starting-value assumed was 410000^{-1} and the final value obtained was 406690^{-1}, which is a little larger (by only 0.45 percent) than the modern accepted value. These results give some idea of the high accuracy achieved by Newcomb, and it is seen that the proportionate error in the Venus mass is not substantially greater than in that of the Earth+Moon.

Next Newcomb discusses the mass of Mercury, and begins by saying that the value "which I have heretofore adopted, $(7{,}500{,}000)^{-1}$, was rather a result of general estimate than of exact computation" (my underline: RAL). For solution (A), derived from the meridian observations alone, Newcomb finally arrives at the value

$$\frac{1 \pm 0.32}{7210000} \qquad \text{solution (A)}$$

while for solution (B), including the observations from transits, he finds

$$\frac{1 \pm 0.35}{7943000} \qquad \text{solution (B)}$$

Newcomb then goes on to discuss the various determinations of the Mercury mass from the motion of Comet Encke that had earlier been made by others, and concludes that the close similarity of the several results with that of Haerdtl based on four equations of LeVerrier is itself a highly suspicious circumstance. Thus he writes, "the consistency of these results seems to be entirely beyo . what the observations are capable of giving, and I hesitate to ascribe much weight to them."

Newcomb next stopped off briefly to attempt a physical approach based on estimating the mean density of the planet. From the fact that the Earth, Venus, and Mars have much the same densities, while the Moon has a density 0.6 that of the Earth, he opined that Mercury probably has a density slightly greater than that of the Moon. Since its angular diameter at 1 AU was known to be 6".6, this would mean a diameter about three-eighths that of the Earth, and then, by assuming a density 0.7 that of the Earth, Newcomb arrived at a mass of about 9000000^{-1} for Mercury. If only he had known how close this was! He then added that "in view of the measured diameter being probably somewhat too small [it was so by about 100 km], these considerations lead us to conclude that the mass is probably <u>between</u> 6000000^{-1} and 9000000^{-1}" (my underline: RAL). But he went no further and did not revert to the discussion again.

Finally he stated that, for the value of the mass to be used in investigating the secular variations, "I have adopted 1.08/(7500000)." It is not made clear just where the factor 1.08 comes from, but it implies a reciprocal mass of 6944444. This is duly tabulated as the value used for Mercury in determining the secular variations, while the values used for Venus and the Earth are given respectively as 406750 and 328000, evidently resulting from small and unimportant rounding-off of the values actually emerging from the earlier discussion.

Having dealt with the secular variations, Newcomb then discussed certain other constants that needed to be reconsidered before final values of the masses could be found. This done, he returned to the question of actual masses, and proceeded to make what he termed a <u>definitive adjustment</u>. In carrying this out, Newcomb first decided that no adjustment of the mass of Mars was necessary, and he went on to consider the weights to be assigned to the value of the solar parallax used, to the constant of aberration, the motion of the node of Venus, and to certain intermediate results. As to the outcome, the value he reached for the mass of Venus was 406600^{-1}. He then reverted to the value hitherto adopted for the solar parallax, which was 8".797, and after brief discussion of its possible range of error finally selected

an intermediate value of 8".790, and this led to the <u>definitive</u> value for Venus of 408000^{-1}. This last adjustment certainly succeeded in bringing the mass nearer to the 1976 value (Table 1-2).

SECTION IX

MASS VALUE AND MEAN DENSITY REALLY GIVEN FOR MERCURY BY NEWCOMB'S INVESTIGATION

Discussion by Newcomb of the mass of Mercury for definitive adjustment begins from the value (B) given earlier herein, namely

$$(1 \pm 0.35)(7943000)^{-1} \qquad \text{solution (B)}$$

Newcomb mentions that the values of the denominator corresponding to the mean-error limits are 5890000 and 12210000. (These are the figures he gives: they should be 5883704 and 12220000; but the differences are not important.) He goes on to say that "these limits are so wide as to include all admissible results for the mass of Mercury," and continues, "moreover, we cannot definitely say that the value of (B), 7943000^{-1}, is markedly greater or less than that given by the weighted mean of all other results, since we might so weight the latter as to give a result greater or less without transcending the bounds of judicious judgment." His next statement on the matter is somewhat less clear, for it says, "I conceive therefore that we are justified in reducing the mean-errors to ±0.26, which will give as the equation of condition, $\nu = -0.055 \pm 0.25$." The quantity ν used here is a parameter that enters in the mass of Mercury in association with a reciprocal mass number in the denominator, thus $\text{mass}^{-1} = (1 + \nu)/\text{number}$, the value of the number taking whatever mass-value is being adjusted. Thus the value of $1 + \nu$ for (B) was found by Newcomb for the purposes of definitive adjustment to be $1 - 0.055 \pm 0.25$, which leads to

$$(0.945 \pm 0.25)(7943000)^{-1} \quad \text{or} \quad (1 \pm 0.26)(8405291)^{-1} \qquad \text{definitive (B)}$$

Strangely enough, with no reason given, this last step in the calculation, which would have yielded the all-desired answer, is nowhere set down in print by Newcomb, but when it is made it shows how the reduced error range, which he does give, namely ±0.26, is related to the value

$\nu = -0.055 \pm 0.25$. Thus the final definitive adjustment for the mass of Mercury leads to a central most-probable value equal to 8405291^{-1}, with a mass-range corresponding to the mean-error limits given by 6670866^{-1} to 11358501^{-1}. Using the presently accepted value of the radius, 2.44×10^{8} cm, this result corresponds to the point X in Figure 2-1 just above the curve at the lower left. On the same basis, the point would lie more or less exactly on the curve for a density of 3.6 g cm^{-3}, with a corresponding reciprocal mass of 9017000

SECTION X

NEWCOMB'S INEXPLICABLE REJECTION OF HIS REAL RESULT IN FAVOR OF A RANDOMLY TAKEN MASS VALUE

It is at the next stage that Newcomb makes a most extraordinary decision. After pointing out, what is unquestionably the case, that it is a practical necessity in producing Tables of the celestial motions to decide upon a specific mass for each planet, he says, "Our Tables must be founded on some perfectly consistent theory, the elements of which shall be so chosen as best to represent the observations." For the mass of Venus, Newcomb duly assigns the adjusted definitive value 408000^{-1}. <u>But then</u>, despite all that has gone before, comes the wholly inexplicable decision

<u>"For the mass of Mercury I took 1 ÷ 6,000,000."</u>

In other words, Newcomb adopted a value not even lying within the error-range of the final value that he had arrived at with such skill and unremitting toil. There is to be found no justification at any point in his discussion for this extraordinary step. He could with equal lack of justification have "taken" 12000000^{-1}!

Where Venus is concerned, the value finally adopted was rounded off to 408000^{-1}, slightly smaller than the figure emerging from the actual solution by a factor 0.9968, and it is remarkably close to the present-day accepted value. On a similar basis for Mercury, the emerging final value would be 8432274^{-1}, while even if it were altered by a factor 0.99, three times the change made for Venus, the mass would only decrease slightly to 8490193^{-1}. To bring it to 6000000^{-1} would require an increase by a factor 1.400882.

Nowhere can there be found any suggested source for so unacceptably large a change at this late stage when only minute adjustments and rounding-offs are being made. Newcomb went on to admit that, "Actually it <u>seems</u> that this mass is larger than the most probable one on either

hypothesis" (my underline:RAL), and he adds, "though not without the range of easy possibility." But what imaginable justification could there be for simply discarding the final central value of 8405291^{-1} so zealously sought and laboriously calculated?

Some minor adjustment (as was made for Venus) to an adjacent round figure, say to 8400000^{-1}, would have been perfectly acceptable and understandable in view of uncertainties. Earlier it was noted herein that Newcomb had described his starting-value of 7500000^{-1} as "rather a result of general estimate than of exact computation." Yet now, when the final result of exact computation is safely in his hands, he rejects it outright in favor of an alien figure well outside the mean-error range. It is entirely against scientific principle and practice to have taken such an unreasonable and indeed quite unnecessary step. In reality, there was no possible choice open to him, after having completed the final definitive adjustment, other than to accept the central value arrived at, namely 8405291, for the reciprocal mass of Mercury. Why at such a stage, when no further decision was called for, should it ever occur to anyone to alter the emergent value to a quite arbitrary, random figure passes comprehension. The labors of many astronomers making over 60,000 observations during a period of more than a century, the extensive work of reducing these observations to suitable form, the organization of expeditions for transit-observations, and the discussion of all the material so obtained by means of intricate planetary theory, with all the concomitant requirements, represented several years work on his own part undertaken with the central objective of finding the masses of Venus and Mercury. Let it be emphasized that the project appears to have been wonderfully successful in determining the mass of Venus with almost surprisingly great accuracy, yet where Mercury is concerned, the vast investigation need never have been carried out, and continued use of the already available general estimate of 7500000 would clearly have been a far better decision.

SECTION XI

CIRCUMSTANCES LEADING TO GENERAL ACCEPTANCE OF THIS ARBITRARY INADMISSIBLE VALUE

In 1896, the year after the publication of The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy, there was held in Paris an International Conference to discuss what values for the planetary masses, and other constants of the solar system, should be generally adopted (Reference 11-1). Newcomb would obviously have had great influence in view of his monumental work on this very matter having been brought to completion in the previous year. In the final session of the conference, Newcomb presented a consistent set of what he regarded as the most probable values for the constants of the solar system, and announced that they had already been introduced into his Tables of the Planets. Curiously enough, however, in the published report of the conference, subtitled Procès-Verbaux, no numerical values whatever for the masses of the planets are recorded. But it must have been the case that Newcomb presented them in some form, in view of their having been adopted after several sessions of discussion. The conference in fact had no official authority whereby to take any decision as binding on others than those present. But the Chairman, with the full consent of the meeting, expressed the hope and assured Newcomb accordingly that his system of constants would become generally adopted with as little delay as possible. He went on to tender the warmest gratitude to Newcomb for having contributed in so important a way to the success of the conference by enabling its members to benefit by the results of his great works.

As to general adoption in the immediate future, clearly the parties principally concerned would have been the Almanac Offices of the Naval Observatory, Washington, and of the Royal Observatory, London, in view of their responsibility for production of the Ephemerides. Although it has not been possible to locate any published record of such agreement, it seems certain that unqualified agreement was promptly reached between these Offices. The constants agreed to be used henceforth included of course the reciprocal mass-values 6000000 for Mercury and 408000 for

Venus. Confirmation of this comes from Newcomb himself, when, in 1903 (Reference 11-2), he wrote that "With the year 1896 came what was perhaps the most important event in my whole plan. Dr. Downing, Superintendent of the British Nautical Almanac, was animated by the same motive", which was to end the confusion that had resulted from the diversity of values of fundamental constants used by astronomers of various institutions. It was these laudable considerations, shared with Newcomb, that led Downing to arrange the 1896 Paris Conference, at which, to quote Newcomb, "it was in fact resolved that beginning with 1901, a certain set of constants should be used in all the emphemerides, substantially the same as those I had worked out" (my underline: RAL).

There was obviously not the smallest element of dissent from this resolution at the conference, which appears to have been conducted in most admirably congenial a manner. But Newcomb relates in his Reminiscences that more than a year later, to his great surprise, a vigorous attack on the work and conclusions of the conference was made by Lewis Boss, who was supported by S. C. Chandler. Their first objection was the rather specious one that the time was not ripe for concluding on a permanent system of astronomical standards, and may have been no more than a screen for their real complaint, which was "that the astronomers of the country should have been consulted before any decision was reached." According to Newcomb, the attack had strange repercussions and re-created much of the earlier confusion, for although foreign ephemerides used "uniform data worked out in the office of the American Ephemeris at Washington for the years beginning 1901, these same data after being partially adopted in the ephemeris for 1900 were thrown out in 1901, and the antiquated ones reintroduced." Newcomb gives no account of how and when the disagreements finally came to an end, but it can be inferred that his views eventually triumphed, for as has been seen his values for the planetary masses came to be firmly entrenched among the I.A.U. System of Constants. It seems possible that the confusion persisted beyond the time of Newcomb's death in 1909, and that steps to restore general harmony were not recommenced until 1911 when there was again held in Paris a congress of those responsible for preparation of national ephemerides, and agreement seems to have been reached on the system of constants for future adoption (Reference 11-3).

SECTION XII

EFFECT OF THE REVISED MERCURY MASS ON THE HELIOCENTRIC COORDINATES OF VENUS

If in fact the true mass of Mercury should be about 9000000^{-1}, it is plain that the perturbations on Venus, which are the largest that Mercury produces on any planet, would be affected in proportion to the mass. The extent to which the secular perturbations of Venus would be affected is not investigated here. However, for the periodic perturbations, although unchanged in periods, they would be reduced in their coefficients. Denoting by L_m and L_v the mean-anomalies of Mercury and Venus, the values of the largest coefficients are for the terms of arguments shown, and are given in Table 12-1 for the two relevant values of the mass.

Table 12-1. Periods and Magnitudes of Largest Perturbations in Longitude of Venus

Argument	Period, yr	6 000 000	9 000 000	Difference
$5L_v - 2L_m$	5.662	0".361	0".241	0".120
$2L_v - L_m$	1.110	0".265	0".177	0".088
$4L_v - 2L_m$	0.555	0".105	0".070	0".035

Since the heliocentric longitudes as published are rounded off to 0".1, such a change of the mass of Mercury could affect the longitude of Venus by a half-range of almost 0".3. The geocentric angular coordinates, however, are given to 0.01 of a second of time (= 0".15) for the R.A., and 0".1 for the Declination. But the angular diameter of Venus, as seen from Earth, ranges from about 10" to 60", and, as earlier noted, measures of the central point of the whole disc are uncertain by about ±1" in both R.A. and Declination. The theoretical changes in the coordinates of Venus that would result from a reduction of the Mercury-mass by one-third may thus be beyond the possibility of optical detection.

For the perturbations by Mercury of the heliocentric distance r of Venus, the three terms of largest coefficients have the same periods as for the longitude, and their values for the two relevant masses are as given in Table 12-2.

Table 12-2. Largest Perturbations in Heliocentric Distance of Venus

Argument	$\frac{\Delta r}{r}$(6 000 000)	$\frac{\Delta r}{r}$(9 000 000)	Reduction $\delta(\Delta r)$, km
$2L_v - L_m$	0".0825	0".0550	14.4
$4L_v - 2L_m$	0".0566	0".0377	9.9
$5L_v - 2L_m$	0".0267	0".0178	4.7

The coefficients of the periodic perturbations Δr would be reduced by the linear amounts $\delta(\Delta r)$ shown in the final column, and as seen range from about 5 to 15 km. As the figures are the semiamplitudes, taken together the terms could yield a total change of about 58 km. Such values may well be capable of detection by means of radar-ranging over a period of a few years.

REFERENCES

1-1. Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, H.M.S.O. London, 1961; and 3rd Impression, 1974.

1-2. Proc. Twelfth Gen. Assembly I.A.U., Hamburg 1964, Vol. XIIB, Academic Press, 1966.

1-3. Proc. Sixteenth Gen. Assembly I.A.U., Grenoble 1976, Vol. XVIB, D. Reidel, 1977.

1-4. Anderson, J. D., Keesey, M. S. W., Lau, E. L., Standish, E. M. Jr., and Newhall, X X, Tests of General Relativity Using Astrometric and Radiometric Observations of the Planets, 3rd International Space-Relativity Symposium, International Astronautical Federation, XXVII Congress 1976.

2-1. Kuiper, G. P., The Planet Mercury: Summary of Present Knowledge, Communs Lunar and Planetary Laboratory, Vol. 8(3), No. 143, Univ. of Arizona, 1970.

2-2. Rabe, E., Derivation of the Fundamental Astronomical Constants from the Observations of Eros 1926-1945, Astron. J. 55, 112, 1950.

2-3. Stracke, G., Abhand. d. Preuss. Akad. d. Wissen., Berlin, 1940.

2-4. Rabe, E., Corrected Derivation of Astronomical Constants from Observations of Eros 1926-1945, Astron. J. 72, 852, 1967.

2-5. Ash, M. E., Shapiro, I. I., and Smith, W. B., Astronomical Constants and Planetary Ephemerides Derived from Radar and Optical Observations, Astron. J. 72, 338, 1967.

2-6. Clemence, G., The System of Astronomical Constants, Ann. Rev. Astron. & Astrophys., 1965.

2-7. Brouwer, D., and Clemence, G., Planets and Satellites, (ed. Kuiper, G. P.), Chicago, 1961.

2-8. Newcomb, S., Elements of the Four Inner Planets and the Fundamental Constants of Astronomy, Washington, 1895.

2-9. Russell, H. N., Astronomy Vol. I, 311 and Appendix ii, Ginn & Co., 1945.

2-10. de Sitter, W., On the System of Astronomical Constants, edited and completed by D. Brouwer after the death of de Sitter (November 1934), Bulletin Astron. Inst. Neth. VIII, 213, 1938.

5-1. Lyttleton, R. A., On the Internal Structures of Mercury and Venus, Astrophys. & Space Sc. 5, 18, 1969.

6-1. Laplace, P. S., Mécanique Céleste, VI, 1803.

6-2. Encke, J. F., Astron. Nach. 443, 1841.

6-3. Rothman, R. W., Memoirs Roy. Astron. Soc. XII, 413, 1842, and M.N.R.A.S. V, 130 and 180, 1843.

6-4. LeVerrier, U. J. J., Ann d. Paris Obs., t. V, Ch. XV, 75 et seq., 1859.

6-5. LeVerrier, U. J. J., Tables of Venus, 1844.

6-6. LeVerrier, U. J. J., Mem. Paris Obs. VI, 92 and 308, 1861.

6-7. von Asten, F. E., Mem. St. Petersburg Akad. XXVI(2), 98 and 109, 1879.

6-8. Tisserand, F., Comptes Rendus XCII, 656, 1881.

6-9. von Haerdtl, E. F., Astron. Nach. 2914, 1889.

6-10. Backlund, J. O., Astron. Nach. 2727, 1885; and Comptes Rendus CXIX, 14, 1894.

6-11. Hill, G. W., Note on the Mass of Mercury, Astron. J. 19, 157, 1898.

6-12. See, T. J. J., On the Probable Mass and Density of Mercury, Astron. Nach. 3743, 1901.

11-1. Conférence Internationale des Étoiles Fondamentales, Procès-Verbaux, Paris, 1896.

11-2. Newcomb, S., Reminiscences of an Astronomer, Harper, 1903.

11-3. Congrès International des Ephemerides Astronomique (1911), Bureau des Longitudes, Paris, 1912.